[4.13] Bode plots

A Bode plot graphs the magnitude and phase angle of a complex function of frequency. Both the frequency and magnitude are plotted on logarithmic axes, but the phase angle uses a linear axis. Bode plots are commonly used in electrical engineering applications to show the performance of filters or other transfer functions. The 89/92+ do not have built-in logarithmic scaling for function plot axes, but you can easily accomplish this effect with a data plot. In general, the steps are:

1. Define the transfer function G(s), where *s* is the frequency-domain variable.
2. Create a frequency list with logarithmic spacing. You can use the function *loglist()* in tip 3.22 to do this, or you can do it manually.
3. Create a list of the magnitudes of the transfer function at each frequency, by applying the built-in *abs()* function to G(s) evaluated at each frequency.
4. Create a list of the phase angles of the transfer function at each frequency, by applying the built-in *angle()* function to G(s) evaluated at each frequency.
5. Create a list of sequential integers, with as many elements as the frequency list in 2. This list will be the *x* variable in the data plot. This list is easily made with the built-in *seq()* function.
6. For the magnitude plot, define a data plot with the lists in 5 and 3.
7. For the phase plot, define a data plot with the lists in 6 and 3.
8. Display the plots with the ZoomData menu item in the Zoom menu.

Other users have written Bode plot programs that are more sophisticated than this method, and result in better-looking plots, but this method is effective if you only need the occasional plot.

As an example, suppose that we want to plot the transfer function, from 10 to 1000 Hz, of a filter defined by

$$G(s) = \frac{w_0 s}{s^2 + \frac{w_0}{Q} s + w_0^2} \qquad \text{where } w_0 = 100 \text{ Hz and } Q = 11$$

This is the general transfer function for a 2nd-order bandpass filter, with unity gain at the center frequency. In general

$$s = jw \quad \text{where} \quad j = \sqrt{-1}$$

(Electrical engineers use *j* for the complex unit, but the TI-89/92+ use *i*).

In the frequency domain, we work with frequencies in radians/second, not Hz, so first we convert the frequencies of interest:

10 Hz = 62.83 rad/sec
100 Hz = 628.3 rad/sec
1000 Hz = 6283 rad/sec

Substituting the values in the transfer function we have

$$G(s)\frac{62.83s}{s^2+57.12s+394761}$$

At the calculator command line, we define this function by entering

```
(62.82*s)/(s^2+57.12*s+394761)→g(s)
```

Make sure that Exact/Approx mode is set to Approx, the angle mode is set to Radian, and the Complex Format is set to Rectangular.

Create the logarithmic frequency list using *loglist()* from tip 3.22. We will plot 50 points between the frequencies of 62 and 6300. Since I have *loglist()* stored in the *\util* folder, the command is

```
util\loglist(62,6300,50)→flist
```

and the list is stored in *flist*. *flist* looks like this: {62, 68, 74.6, 81.8, ... 4774, 5237, 6300}

To create the list of transfer function magnitudes at each frequency, use

```
20*log(abs(g(i*w)))|w=flist→mag
```

The expression 20*log() converts the magnitudes to decibels, which are typically used to express transfer function gain. The built-in *abs()* function is used to find the magnitude of the complex result of the transfer function, and *i* is the complex unit, not the letter "i". The magnitudes are stored in the list *mag*.

To create the list of transfer function phase angles, use

```
angle(g(i*w))*(180/π)|w=flist→phase
```

The expression `*(180/π)|` is used to convert the phase angles from radians to degrees. Omit it if you want to plot the phase angle in radians. The resulting list is stored in the variable *phase*.

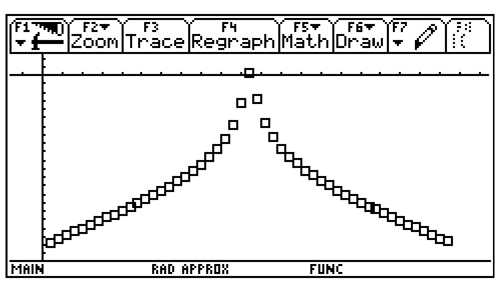
To create the list of sequential integers, use

```
seq(k,k,1,51)→xcoord
```

Note that the number of elements, 51, is one more than the *loglist()* argument of 50, since *loglist()* returns one more element than the number specified.

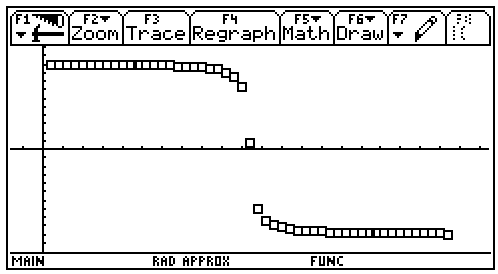
Now that we have the three lists, it is a simple matter to use the built-in data plotting functions to create the plots. Use these keystrokes to set up and display the magnitude plot:

1. Press [Y=] to display the Y= editor
2. Use [UP] and [DOWN] to highlight Plot 1
3. Press [F3] to edit the plot definition.
4. Set the Plot Type to Scatter, set the Mark to Box, set *x* to *xcoord*, and set *y* to *mag*. Push [ENTER], [ENTER] to finish editing.
5. Press [F2] to display the Zoom menu, then press [9] to select ZoomData. This is the resulting magnitude plot on a 92+:



The procedure to display the phase plot is similar:

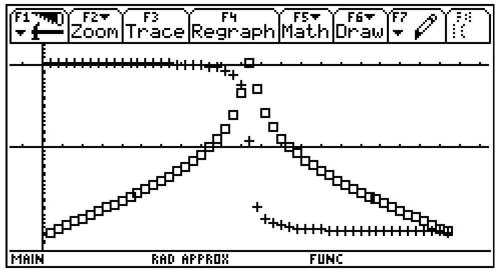
1. Press [Y=] to display the Y= editor
2. Use [UP] and [DOWN] to highlight Plot 2
3. Press [F3] to edit the plot definition.
4. Set the Plot Type to Scatter, set the Mark to Box, set *x* to *xcoord*, and set *y* to *phase*. Push [ENTER], [ENTER] to finish editing.
5. Use [UP] and [DOWN] to highlight the definition for Plot 1, then press [F4] to remove the check mark.
6. Press [F2] to display the Zoom menu, then press [9] to select ZoomData. This is the resulting phase plot on a 92+:



For both the magnitude and phase plots, you can use [F3] to trace the graph and read either the magnitude or the phase at the bottom of the display, as the *yc* variable. However, you cannot directly read the frequency, since the *xc* variable only shows the elements of the *xcoord* list. You can, though, manually display the frequency. Suppose that we want to know the lower frequency at which the magnitude is about -30dB. Push [F3] to trace the plot, then use [LEFT] and [RIGHT] to move the cursor until *yc* is about -30. For this example, *yc* is -29.67 when *xc* is 13. Press [HOME] to return to the Home screen, then enter flist[13] in the entry line. The result is about 188 rad/sec, or 30 Hz.

It is also possible to show the magnitude and phase plots on the same screen, which is useful for determining the relationship between the two results. Use these steps:

1. Create the magnitude plot as described above.
2. Press [F1] to display the Tools menu, then press [2] to select Save Copy As. In the dialog box, set the Type to Picture, and set the Variable name to *magplot*. Press [ENTER], [ENTER] to close the dialog box.
3. Press [Y=] to display the Y= editor. Select Plot 1 (the magnitude plot) and press [F4] to remove the check mark.
4. Create the phase plot as Plot 2, as described above. You may want to use a different symbol, such as the '+' symbol, so that you can easily distinguish the two plots. Display the phase plot by pressing [F2], then [9] to select ZoomData.
5. With the phase plot still shown, press [F1] to open the Tools menu, then press [1] to select Open. In the dialog box, set the Type to Picture, and select *magplot* as the Variable. Push [ENTER], [ENTER] to close the dialog box. The magnitude plot is shown superimposed on the phase plot, like this:



Here are some more tips on using this method:

- The number of elements in *flist* determines the plot resolution. More elements result in a better-looking plot, but it takes longer to calculate the magnitude and phase. There isn't much point to using more than 240 elements on a 92+, or 160 elements on the 89, since those are the screen widths in pixels. If you use a lot of elements, choose Dot or Box for the symbol type. However, for a quick graph with only a few points, the Box symbol clearly shows each point, even if the points lie on an axis.
- It is sometimes instructive to plot the phase of the transfer function, as a function of magnitude. This is easily accomplished once you have created the *mag* and *phase* lists. Simple create a data plot definition with *mag* as the *x*-variable, and *phase* as the *y*-variable.

Biographical note

The Bode plot is named for Hendrik Bode, an engineer and mathematician working at Bell Laboratories in the 1920's and 1930's. Along with Harold Black and Harry Nyquist, Bode was instrumental in developing a comprehensive regenerative feedback theory. He invented techniques for synthesizing networks with desired characteristics, which has applications in telephone line equalization. He also worked with R. B. Blackman and Claude Shannon to model messages and signals in probabilistic terms, which was a new idea at the time.

Hendrik Bode is obviously not the other well-know Bode, the German astronomer Johann Elert Bode, who lived from 1747 to 1826. J.E. Bode compiled an astronomical atlas for 51 consecutive years, and also, unfortunately, his name is associated with a flawed theory to predict the location of planetary orbits.