[6.32] Accurate approximate solutions to quadratic equations with large coefficients

This tip shows a method to calculate the approximate roots of the quadratic equation $ax^2 + bx + c =$, when *a* and *b* are very small. In this case, calculating the roots with the 'classical' solution formula results in less accurate roots. The classical solution is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{or} \qquad x = \frac{2c}{-b \pm \sqrt{b^2 - 4ac}}$$

These solutions result in round-off errors for small values of *a* and *b.* A better solution is

$$q = \left(-\frac{1}{2}\right)\left(b + \text{sign}(b)\sqrt{b^2 - 4ac}\right) \quad \text{then}$$

$$x1 = \frac{q}{a} \qquad \text{and} \qquad x2 = \frac{c}{q}$$

This function, *quadrtic(),* uses these equations.

```
quadrtic(aa,bb,cc)
func
©(a,b,c) in ax^2+bx+c=0
©27oct00 dburkett@infinet.com
local q

when(bb≠0,-(bb+sign(bb)*√(bb*bb-4*aa*cc))/2,-(bb+√(bb*bb-4*aa*cc))/2)→q
{q/aa,cc/q}

Endfunc
```

To use *quadrtic(),* call it with the coefficients (a,b,c). The two roots are returned as a list. For example,

`quadrtic(8,-6,1)` returns {0.5,0.25}

To put either of the roots in the history display, use a list index of [1] or [2] as needed:

{0.5,0.25}[1] puts 0.5 in the history display
{0.5,0.25}[2] puts 0.25 in the history display

The TI92 operation mode should be set to APPROXIMATE, since the purpose of *quadrtic()* is to reduce floating-point arithmetic errors. If you need exact answers, the classical solution equations are just as good. If *quadrtic()* is run with the mode set to EXACT, it may take a very, very long time to return the answers.

For example, let a = 1E-400, b = -6 and c = 1. The classic solution gives

x1 = 6E400 y1 = 1
x2 = 0 y2 = 1

The improved solution equations give

x1 = 6E400 y1 = 1
x2 = 0.1666... y2 = 0

The improved method gives a better root for x2.

Note that the built-in *zeros()* function on the TI92+ gives the same results as the classical method.

quadrtic() will return complex results if the complex format is set to RECTANGULAR or POLAR. Otherwise, equations with complex results will result in an execution error.

quadrtic() returns these solutions for combinations of a=0, b=0 and c=0:

a=0	root 1 undefined, root 2 is solution to bx+c=0
b=0	two identical real roots, or 2 conjugate roots
c=0	one root is zero
a=0 and b=0	both roots are undefined; returns {undef undef}
b=0 and c=0	returns 0 and 'undef'
a=0 and b=0 and c=0	both roots are undefined