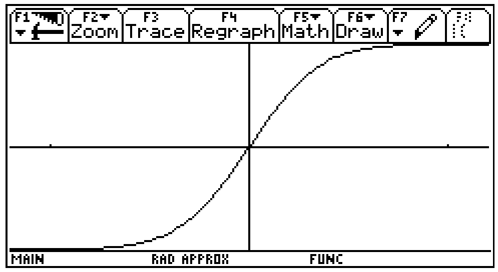
[6.34] Error function for real arguments

The error function is one of many so-called 'special functions'. It is defined as

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

For x > -3 and x < 3, the error function looks like this:



erf(x) approaches 1 and -1 for large x and large -x, respectively. Also note that erf(-x) = -erf(x). The complimentary error function erfc(x) is defined as

erfc(x) = 1 - erfc(x)

This function calculates erf(x) for all real *x*.

```
erf(x)
func
©(x) error function of x
©dburkett@infinet.com 13 nov 99
©x real and -1E-999 <= x <= 1E999
©Error +/- 1E-14.

when(x<0,-erf(-x),when(x<.84375,x+x*polyeval({-2.37630166566650E-5,-5.77027029648
94E-3,-2.8481749575599E-2,-3.25042107247E-1,1.2837916709551E-1},x*x)/polyeval({-
3.9602282787754E-6,1.3249473800432E-4,5.0813062818758E-3,6.5022249988767E-2,3.97
91722395916E-1,1},x*x),when(x<1.25,.84506291151047+polyeval({-2.1663755948688E-3
,3.5478304325618E-2,-1.108946942824E-1,3.1834661990116E-1,-3.722078760357E-1,4.1
485611868375E-1,-2.3621185607527E-3},x-1)/polyeval({1.1984499846799E-2,1.3637083
912029E-2,1.2617121980876E-1,7.1828654414196E-2,5.4039791770217E-1,1.06420880400
84E-1,1},x-1),when(x<2.8571428571429,1-1/x*e^(-x*x-.5625+polyeval({-9.8143293441
691,-8.1287435506307E1,-1.8460509290671E2,-1.6239666946257E2,-6.2375332450326E1,
-1.05586262253234E1,-6.9385857270718E-1,-9.8649440348471E-3},1/x^2)/polyeval({-6
.0424415214858E-2,6.5702497703193,1.0863500554178E2,4.2900814002757E2,6.45387271
73327E2,4.3456587747523E2,1.3765775414352E2,1.9651271667439E1,1},1/x^2)),when(x<
5.518,1-1/x*e^(-x*x-.5625+polyeval({-4.8351919160865E2,-1.0250951316111E3,-6.375
6644336839E2,-1.6063638485582E2,-1.7757954917755E1,-7.9928323768052E-1,-9.864942
9247001E-3},1/x^2)/polyeval({-2.2440952446586E1,4.7452854120696E2,2.553050406433
2E3,3.1998582195086E3,1.5367295860844E3,3.2579251299657E2,3.0338060743482E1,1},1
/x^2)),1)))))

Endfunc
```

This function is accurate to full machine precision for all arguments. The algorithm is taken from module ERF in the FDLIBM package. This link provides the algorithm details:

http://gams.nist.gov/serve.cgi/Module/FDLIBM/ERF/13299/

While the function may look complicated, it is really nothing more than a series of nested *when()* functions to select the appropriate estimating method for value the input *x* argument. *erf()* calls itself for x<0; there is only one level of recursion.

These references have more discussion of the error function:

Handbook of Mathematical Functions, Abramowitz and Stegun, Dover, 1965.

Special Functions and Their Applications, Lebedev, Dover, 1972.