

[6.48] Use R→Pθ() for four-quadrant arc tangent function

The built-in arc tangent function, $\tan^{-1}()$, cannot return the correct quadrant of an angle specified by x- and y-coordinates, because the argument does not contain enough information. Suppose we want the angle between the x-axis and a ray with origin (0,0) and passing through point (1,1). Since

$$\tan(\theta) = \frac{y}{x} \quad \text{we have} \quad \tan(\theta) = \frac{1}{1} \quad \text{or} \quad \theta = \tan^{-1}(1) \quad \text{so} \quad \theta = \frac{\pi}{4}$$

However, if the ray instead passes through (-1,-1), we get the same result since (-1/-1) is also equal to 1, but the angle is actually $(-3\pi)/4$. This difficulty was addressed by the designers of the Fortran programming language, which includes a function called $\text{atan2}(y,x)$ to find the arc tangent of y/x correctly in any of the four quadrants, by accounting for the signs of x and y.

It is a simple matter to accomplish this in TI Basic with the built-in $R \rightarrow P\theta()$ function, if we account for the special case (0,0). In fact, it can be done with a single line of TI Basic code:

```
when(x=0 and y=0,undef,R→Pθ(x,y))
```

undef is returned for (0,0), otherwise $R \rightarrow P\theta(0,0)$ returns itself in Radian angle mode, and this expression in Degree angle mode:

```
180*R→Pθ(0,0)/π
```

Neither of these results are useful.

A more elaborate function can be written which also handles list and matrix arguments:

```
atan2(ax,ay)
Func
@(x,y) 4-quadrant arctan(y/x)
©Must be installed in math\
©6jan02/dburkett@infinet.com

local at,em,tx                                     © Function name, error message, ax type

"atan2 error"→em                                   © Initialize error message

define at(α,β)=func                                  © Function finds atan2() of simple elements
  when(α=0 and β=0,undef,R→Pθ(α,β))
endfunc

getType(ax)→tx                                     © Save argument type for later tests

if tx≠getType(ay):return em                         © Return error if arguments not same type

if tx="LIST" then                                    © Handle list arguments
  if dim(ax)≠dim(ay):return em
  return seq(at(ax[k],ay[k]),k,1,dim(ax))

elseif tx="MAT" then                                © Handle matrix arguments
  if rowdim(ax)≠rowdim(ay) or coldim(ax)≠coldim(ay):return em   © Validate dimensions
  return list→mat(math\atan2(mat→list(ax),mat→list(ay)),coldim(ax))

elseif tx="NUM" then                                © Handle numeric arguments
  return at(ax,ay)

else                                                © Return error for all other arg types
  return em

endif

EndFunc
```

Both arguments of *atan2()* must be the same type, and must be numbers, lists or expressions. *atan2()* does not work with symbolic arguments.

Typical calls and results in Degree angle mode are:

| | | |
|-----------------------------------|---------|-----------|
| <code>atan2(1,1)</code> | returns | 45 |
| <code>atan2(-1,-1)</code> | returns | -135 |
| <code>atan2({1,-1},{1,-1})</code> | returns | {45,-135} |

| | | | |
|--------------------|---|---------|--|
| <code>atan2</code> | $\left(\left[\begin{array}{cc} 1 & -1 \\ 0 & 3 \end{array} \right], \left[\begin{array}{cc} 1 & 1 \\ 0 & \sqrt{3} \end{array} \right] \right)$ | returns | $\left[\begin{array}{cc} 45 & 135 \\ \text{undef} & 30 \end{array} \right]$ |
|--------------------|---|---------|--|